Readers' Forum

Comment on "New Eddy Viscosity Model for Computation of Swirling Turbulent Flows"

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IN a recent paper, Kim and Chung[1] develop an improved k-ϵ model for swirling flows by formally operating on the algebraic Reynolds stress model proposed by Rodi.[2] This is done after Rodi's model is first written in terms of alternate coefficients as

$$\frac{\overline{u_i u_j}}{k} = \phi_1 \frac{P_{ij}}{\epsilon} + \phi_2 \delta_{ij} \tag{1}$$

where

$$\phi_1 = \frac{1-c_2}{(P/\epsilon)+c_1-1} \tag{2a}$$

$$\phi_2 = \frac{2}{3}\bigg/\frac{c_2(P/\epsilon)+c_1-1}{(P/\epsilon)+c_1-1} \tag{2b}$$

By assuming weakly swirling flow, the authors show that the dominant turbulent shear stress components can be expressed as

$$-\overline{uv} = \nu_t \frac{\partial U}{\partial r} \tag{3a}$$

$$-\overline{vw} = \nu_t \frac{\partial W}{\partial r} \tag{3b}$$

where ν_t is a Richardson number-dependent eddy viscosity, namely

$$\nu_t = \frac{\alpha}{1+\beta Ri}\frac{k^2}{\epsilon} \tag{4}$$

with

$$Ri = \frac{k^2}{\epsilon^2}\frac{W}{r}\frac{\partial W}{\partial r}, \qquad \alpha = \phi_1\phi_2, \qquad \beta = 4\phi_1^2 \tag{5}$$

For the case of vanishing Ri, the authors correctly note that α must be equal to C_μ, whose specified numerical value (0.09) applies for a flow in local equilibrium ($P/\epsilon = 1$). The authors then select a value for β ($= 0.25$) by stating that β should be in the range $0.05 \le \beta \le 0.44$ for a flow in local equilibrium. On the basis of Eq. (5), this set of values ($\alpha = 0.09$, $\beta = 0.25$) corresponds to $\phi_1 = 0.25$ and $\phi_2 = 0.36$. These values for ϕ_1 and ϕ_2 are not, however, compatible with each other. This can be shown by letting $P/\epsilon = 1$ in Eq. (2) and noting that

$$\phi_2 = \frac{2}{3}\bigg/\frac{c_2+c_1-1}{c_1} = \frac{2}{3}\bigg/\left[1-\frac{(1-c_2)}{c_1}\right] = \frac{2}{3}\bigg/(1-\phi_1)$$

so that

$$3\phi_2 = 2(1-\phi_1) \tag{6}$$

which is *not* satisfied when $\phi_1 = 0.25$ and $\phi_2 = 0.36$. This equation also can be generated directly from Eq. (1) by letting $i = j$, and noting that $\overline{u_i u_i}/k = 2$, $P_{ii}/\epsilon = 2(P/\epsilon) = 2$, and $\delta_{ii} = 3$. Since the authors' values for ϕ_1 and ϕ_2 do not satisfy Eq. (6), their specified values also violate the constraint that $\overline{u_i u_i}/k = 2$. From the point of view of predicting mean-flow behavior in swirling free jet flows, violation of this constraint is not serious, but if the authors' model (or some extension thereof that includes wall effects of local pressure-strain behavior) is also to apply to more complicated swirling flows where normal stress effects are important, then specification of numerical values for ϕ_1 and ϕ_2 must be done with greater care. A more rigorous approach would be to substitute C_μ/ϕ_1 for ϕ_2 in Eq. (6), which then yields a quadratic equation for ϕ_1. The solution of this equation can be written as

$$\phi_1 = \frac{1 \pm (1-6C_\mu)^{1/2}}{2} \tag{7}$$

from which $\phi_1 = 0.16$ (negative root) and $\phi_2 = 0.56$ when $C_\mu = 0.09$. These values for ϕ_1 and ϕ_2 satisfy the constraint $\overline{u_i u_i}/k = 2$, whereas the authors' values do not ($\overline{u_i u_i}/k = 1.72$ when $\phi_1 = 0.25$ and $\phi_2 = 0.36$).

It should be emphasized here that if the above approach is followed, then the numerical value of C_μ *alone* determines all coefficient values in the authors' model, including the coefficient β that models swirl effects in the flow [refer to Eq. (4)]. This approach represents a distinct improvement (at least in concept) over models in which coefficient values are specified by referring separately to stress levels measured in plane homogeneous shear flow (PHSF) and to critical Richardson number behavior (e.g., the manner in which the ϕ and ϕ' coefficients are specified in the Gibson and Younis[3] model).

In this context it should be noted that only those values of c_1 and c_2 that yield $\phi_1 = 0.16$ and $\phi_2 = 0.56$ will simultaneously satisfy the condition that $C_\mu = \phi_1\phi_2 = 0.09$ when $P/\epsilon = 1$. If, for example, the values for c_1 and c_2 specified by Gibson and Younis[3] are introduced into Eq. (2), then $\phi_1 = 0.23$ and $\phi_2 = 0.51$. Similarly, if $c_1 = 1.5$ and $c_2 = 0.6$, as prescribed by Launder, Reece, and Rodi,[4] then $\phi_1 = 0.27$ and $\phi_2 = 0.49$. Both sets of values for ϕ_1 and ϕ_2 satisfy the constraint given by Eq. (6) and yield stress levels that approximate those measured in PHSF by Champagne, Harris, and Corrsin.[5] They do not, however, satisfy the condition that $C_\mu = \phi_1\phi_2 = 0.09$ but yield elevated values instead ($C_\mu = 0.12$: GY; $C_\mu = 0.13$: LRR). The numerical value for ϕ_1 calculated from Eq. (7), namely 0.16, is admittedly somewhat less than typical values of the ratio $(1-c_2)/c_1$ corresponding to previously proposed values for c_1 and c_2 (cf. Table 1 of Ref. 3), noting that ϕ_1 and $(1-c_2)/c_1$ should be equal when $P/\epsilon = 1$ on the basis of Eq. (2a). This is not a point of concern, however, inasmuch as calculated stress levels still closely approximate those measured by Champagne, Harris, and Corrsin[5] in PHSF. This can be shown by considering the expanded form of Eq. (1) that applies for PHSF in the xy plane, namely

$$\frac{\overline{u^2}}{k} = \frac{2}{3}\bigg/(1+2\phi_1), \quad \frac{\overline{v^2}}{k} = \frac{\overline{w^2}}{k} = \phi_2, \quad \frac{-\overline{uv}}{k} = (\phi_1\phi_2)^{1/2} \tag{8}$$

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For the suggested revised coefficient values ($\phi_1 = 0.16$, $\phi_2 = 0.56$),

$$\frac{\overline{u^2}}{k} = 0.88, \quad \frac{\overline{v^2}}{k} = \frac{\overline{w^2}}{k} = 0.56, \quad \frac{-\overline{uv}}{k} = 0.30$$

which agree reasonably well with the normalized stress levels measured by Champagne, Harris, and Corrsin ($\overline{u^2}/k = 0.93$, $\overline{v^2}/k = 0.48$, $\overline{w^2}/k = 0.59$, $-\overline{uv}/k = 0.33$).

Finally, it should be noted that if the suggested alternate value for ϕ_1 is adopted, then β will be lower by a factor 2.5 than the value of β specified by the authors. In reference to Eq. (4), this change will diminish the effect of swirl on the eddy viscosity and, in turn, will alter predictions in comparison to those shown in Figs. 1-3 of Ref. 1. It would be informative if the authors were to repeat their calculations in order to demonstrate the performance of their model when referred to a value of β that is internally consistent with the overall model.

References

[1]Kim, K. Y. and Chung, M. K., "New Eddy Viscosity Model for Computation of Swirling Turbulent Flows," *AIAA Journal*, Vol. 25, July 1987, pp. 1020–1022.

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Comment on "New Eddy Viscosity Model for Computation of Swirling Turbulent Flows"

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WHILE the authors may be right that c_1 and c_2 vary greatly in the literature, these constants are not independent but should obey

$$\frac{1 - c_2}{c_1} = \phi_1 \approx 0.22$$

when $P/\epsilon = 1$. Note that this gives $\beta = 4\phi_1^2 = 4(0.22)^2 = 0.19$, so you are not at liberty to take $\beta = 0.25$, otherwise you implicitly modify c_1 and/or c_2 in an arbitrary ad hoc fashion.

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Reply by Authors to F. B. Gessner and M. A. Leschziner

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WE very much appreciate Drs. Gessner and Leschziner for their comments and private advice about our k-ϵ model for weakly swirling turbulent flows.[1]

They seem to assert that the ratio P/ϵ must be taken to be an equilibrium value of unity in order to determine the model constant β in Eq. (5) of Dr. Gessner's comment. Such practice, however, is permitted only for convenience and simplicity in determining unknown model constants. Rather, we believe that it is more desirable and accurate to use a representative value of P/ϵ, if available, in the flowfield under consideration.

In reality, P/ϵ vanishes at both the center and near free boundary of the swirling jet, and $0 < P/\epsilon \leq 1$ in the rest of the flowfield. This means that $P/\epsilon = 1$ is not a good selection to represent the total flowfield.

Our model constant $\beta = 0.25$ implicitly assumes that P/ϵ is about 0.8 for $c_1 = 1.8$ and $c_2 = 0.6$[2] or $c_1 = 3.$ and $c_2 = 0.3$.[3] And if $c_1 = 2.2$ and $c_2 = 0.55$,[4] $\beta = 0.25$ implies that $P/\epsilon = 0.6$. In this respect, since $(1 - P/\epsilon) \geq 0$, the constant value of $(1 - c_2)/c_1$ as, say, 0.22 in Leschziner's comment constrains the lower asymptote of β [note that $\beta = 4\phi_1^2$ where ϕ_1 is in Eq. (2) of Gessner's comment]. The accompanying figure shows comparison of model performances with varying β, which suggests that $\beta = 0.25$ is a best choice.

As a conclusion, we would like to mention that it is usual practice to select the best model constant with reference to available data after theoretical derivation of a physical model.

References

[1]Kim, K. Y. and Chung, M. K., "New Eddy Viscosity Model for Computation of Swirling Turbulent Flows," *AIAA Journal*, Vol. 25, July 1987, pp. 1020–1022.

[2]Gibson, M. M. and Launder, B. E., "Ground Effects on Pressure Fluctuations in the Atmospheric Boundary Layer," *Journal of Fluid Mechanics*, Vol. 86, 1978, pp. 491-511.

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[4]Launder, B. E., "On the Effects of a Gravitational Field on the Turbulent Transport of Heat and Momentum," *Journal of Fluid Mechanics*, Vol. 67, 1975, pp. 569–581.

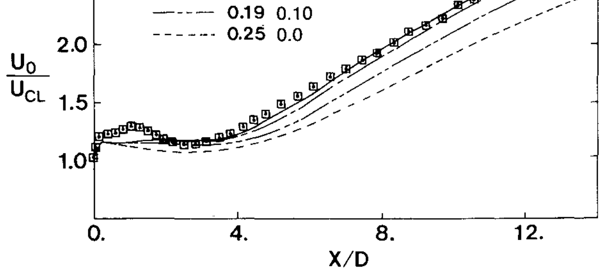

Fig. 1 Comparison of predicted decay of the centerline velocity of a coaxial swirling jet with different values of β.

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